MA1025 (4-0) Finite Mathematics for Operations Research Objectives

Following is an *illustrative* list of objectives for MA1025. The successful student will be able to synthesize solutions to problems not explicitly indicated in this list. It is assumed that the entering student has at some point mastered the elementary operations of college algebra by completing such a course as an undergraduate, and that the student is responsible for reviewing the subject as necessary.

1. Propositional and Predicate Logic

- (a) Decide whether a given statement is a proposition in the logical sense.
- (b) State the definitions of the fundamental logical connectives: negation, conjunction, disjunction, and implication.
- (c) Use truth tables to prove simple logical equivalences and logical implications.
- (d) State the laws of logic (commutativity, associativity, DeMorgan's laws, etc.).
- (e) Apply the laws of logic to the proof of logical equivalences.
- (f) Explain the use of the existential and universal quantifiers.
- (g) Construct the negation of a proposition using one or more quantifiers.
- (h) Be able to state, and use, a small set of logical inference rules.
- (i) Describe in logical terms the fundamental techniques of proof in mathematics.
- (j) Use the Principle of Mathematical Induction to prove simple algebraic identities.

2. Sets and Set Operations

- (a) Describe the different notations that are commonly used to represent sets.
- (b) Distinguish between set membership and set inclusion.
- (c) State the definitions of the fundamental set operations: union, intersection, complement, relative complement.
- (d) For each of the laws of propositional logic, state the set-theoretic equivalent, and apply these laws to the construction of equational proofs of set identities.
- (e) Given sets A and B, prove that A is a subset of B by (a) using an "element-chasing" argument and (b) applying the set-generating notation together with the laws of propositional logic.
- (f) Demonstrate the use of indexing to represent families of sets.
- (g) Give an example of a power set.

(h) Describe the Cartesian product of two sets.

3. Functions and Relations

- (a) Explain the idea of a relation from a set A to a set B.
- (b) Classify a given relation on a set by determining whether it is reflexive, symmetric, antisymmetric, and/or transitive.
- (c) Give examples of equivalence relations and partial order relations.
- (d) Explain the idea of a function with domain A and codomain B.
- (e) Given a function $f:A\to B$, determine whether f is an injection, a surjection, or a bijection.
- (f) Given $f:A\to B$ and $g:B\to C$, describe $g\circ f:A\to C$ and determine its properties.

4. Counting

- (a) Explain what is meant by the cardinality of a finite set.
- (b) Explain what it means for two infinite sets to have the same cardinality.
- (c) State the addition and product rules for counting.
- (d) State the definition of permutation.
- (e) State the definition of r-permutation, and give a formula for computing the number of r-permutations of an n-set.
- (f) State the definition of r-combination, and give a formula for computing the number of r-combinations of an n-set.
- (g) State the binomial theorem.
- (h) Apply the binomial theorem to determine the coefficients on each term in the expansion of $(a + b)^n$ for specified a, b, and n.
- (i) Apply the binomial theorem, when applicable, to prove specified identities.
- (j) State the multinomial theorem.
- (k) Apply the binomial theorem to determine the coefficients on each term in the expansion of $(a_1 + a_2 + \cdots + a_k)^n$ for specified a_1, a_2, \ldots, a_k , and n.